Class X

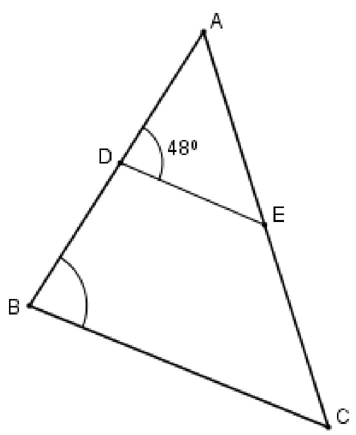
Mathematics

Sample Question Paper 2018-19

Time allowed: 3 Hours **Max. Marks: 80**

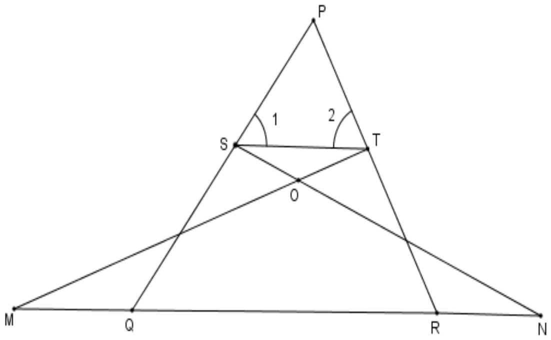
General Instructions:

1. All the questions are compulsory.
2. The questions paper consists of 30 questions divided into 4 sections A, B, C and D.
3. Section A comprises of 6 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 10 questions of 3 marks each. Section D comprises of 8 questions of 4 marks each.
4. There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted.

	Section-A	
1.	Find the value of a, for which point P ($\frac{a}{3}$, 2) is the mid-point of the line segment joining the points Q(-5,4) and R(-1,0).	**1**
2.	Find the value of k, for which one root of the quadratic equation kx^2-14x+8 = 0 is 2.	**1**
	OR	
	Find the value(s) of k for which the equation $x^2 + 5kx + 16 = 0$ has real and equal roots.	
3.	Write the value of $\cot^2\theta - \frac{1}{\sin^2\theta}$	**1**
	OR	
	If $sin\theta = cos\theta$, then find the value of $2\tan\theta + \cos^2\theta$	
4.	If nth term of an A.P. is (2n+1), what is the sum of its first three terms?	**1**
5.	In figure if AD= 6cm, DB=9cm, AE = 8cm and EC = 12cm and ∠ADE = 48^0. Find ∠ABC 	**1**
6.	After how many decimal places will the decimal expansion of $\frac{23}{2^4 \times 5^3}$ terminate?	**1**

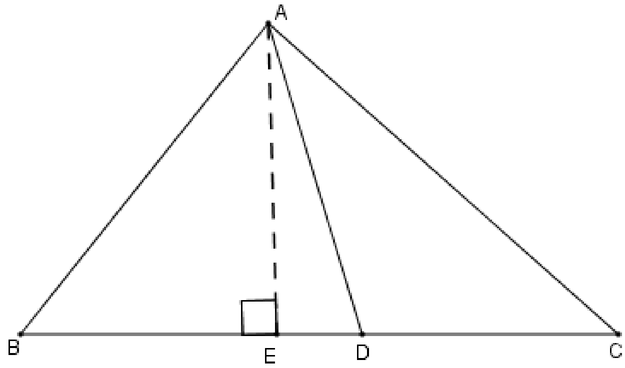
Section-B		
7.	The HCF and LCM of two numbers are 9 and 360 respectively. If one number is 45, find the other number.	**2**
	OR	
	Show that $7-\sqrt{5}$ is irrational, give that $\sqrt{5}$ is irrational.	
8.	Find the 20^{th} term from the last term of the AP 3,8,13,….,253	**2**
	OR	
	If 7 times the 7^{th} term of an A.P is equal to 11 times its 11^{th} term, then find its 18^{th} term.	
9.	Find the coordinates of the point P which divides the join of A(-2,5) and B(3,-5) in the ratio 2:3	**2**
10.	A card is drawn at random from a well shuffled deck of 52 cards. Find the probability of getting neither a red card nor a queen.	**2**
11.	Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is a prime number	**2**
12.	For what value of p will the following pair of linear equations have infinitely many solutions $(p-3)x+3y = p$ $px+py = 12$	**2**
Section-C		
13.	Use Euclid's Division Algorithm to find the HCF of 726 and 275.	**3**
14.	Find the zeroes of the following polynomial: $5\sqrt{5}x^2+30x+8\sqrt{5}$	**3**
15.	Places A and B are 80 km apart from each other on a highway. A car starts from A and another from B at the same time. If they move in same direction they meet in 8 hours and if they move towards each other they meet in 1 hour 20 minutes. Find the speed of cars.	**3**
16.	The points A(1,-2) , B(2,3), C (k,2) and D(-4,-3) are the vertices of a parallelogram. Find the value of k.	**3**
	OR	
	Find the value of k for which the points (3k-1,k-2), (k,k-7) and (k-1,-k-2) are collinear.	
17.	Prove that $\boldsymbol{\cot\theta - \tan\theta = \frac{2\cos^2\theta-1}{\sin\theta\cos\theta}}$	**3**
	OR	
	Prove that $\boldsymbol{\sin\theta(1+\tan\theta)+\cos\theta(1+\cot\theta) = \sec\theta + \mathrm{cosec}\theta}$	
18.	The radii of two concentric circles are 13 cm and 8 cm. AB is a diameter of the bigger circle and BD is a tangent to the smaller circle touching it at D and intersecting the larger circle at P on producing. Find the length of AP.	**3**

19. In figure $\angle 1 = \angle 2$and $\Delta NSQ \cong \Delta MTR$, then prove that $\Delta PTS \sim \Delta PRQ$. **3**



OR

In ΔABC, if AD is the median, then show that $AB^2+AC^2 = 2(AD^2+BD^2)$



20. Find the area of the minor segment of a circle of radius 42cm, if length of the corresponding arc is 44cm. **3**

21. Water is flowing at the rate of 15 km per hour through a pipe of diameter 14cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tank will rise by 21 cm. **3**

OR

A solid sphere of radius 3 cm is melted and then recast into small spherical balls each of diameter 0.6cm. Find the number of balls.

22. The table shows the daily expenditure on grocery of 25 households in a locality. Find the modal daily expenditure on grocery by a suitable method. **3**

Daily Expenditure (in Rs.)	100-150	150-200	200-250	250-300	300-350
No of households	4	5	12	2	2

Section-D

23.	A train takes 2 hours less for a journey of 300km if its speed is increased by 5 km/h from its usual speed. Find the usual speed of the train. **OR** Solve for x: $\frac{1}{(a+b+x)} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$, $[a \neq 0, b \neq 0, x \neq 0, x \neq -(a+b)]$	**4**
24.	An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.	**4**
25.	Prove that in a right angled triangle square of the hypotenuse is equal to sum of the squares of other two sides.	**4**
26.	Draw a ΔABC with sides 6cm, 8cm and 9 cm and then construct a triangle similar to ΔABC whose sides are $\frac{3}{5}$ of the corresponding sides of ΔABC.	**4**
27.	A man on the top of a vertical observation tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30^0 to 45^0, how long will the car take to reach the observation tower from this point? **OR** The angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30^0 and the angle of depression of its shadow from the same point in water of lake is 60^0. Find the height of the cloud from the surface of water.	**4**
28.	The median of the following data is 525. Find the values of x and y if the total frequency is 100.	**4**

Class Interval	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	Y
700-800	9
800-900	7
900-1000	4

OR

The following data indicates the marks of 53 students in Mathematics.

Marks	Number of students
0-10	5
10-20	3
20-30	4
30-40	3
40-50	4
50-60	4
60-70	7
70-80	9
80-90	7
90-100	8

Draw less than type ogive for the data above and hence find the median.

29. The radii of circular ends of a bucket of height 24 cm are 15 cm and 5 cm. Find the area of its curved surface. **4**

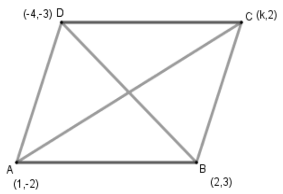
30. If $sec\theta + tan\theta = p$, then find the value of $cosec\theta$. **4**

Class: X
Mathematics
Marking Scheme 2018-19

Time allowed: 3hrs **Maximum Marks: 80**

Q No	SECTION A	Marks
1	$\left(\frac{-5+(-1)}{2}, \frac{4+0}{2}\right) = \left(\frac{a}{3}, 2\right)$ $\frac{a}{3} = \frac{-6}{2} \Longrightarrow a = -9 \Longrightarrow$	1
2	4K -28+8 = 0 K= 5	$1/_2$ $1/_2$
	OR	
	For roots to be real and equal, $b^2 - 4ac = 0$ $\Longrightarrow (5k)^2 - 4 \times 1 \times 16 = 0$ $k = \pm\frac{8}{5}$	$1/_2$ $1/_2$
3	$cot^2\theta - \frac{1}{\sin^2\theta} = cot^2\theta - cosec^2\theta$ = -1	1 $1/_2$ $1/_2$
	OR	
	$\sin\theta = \cos\theta \quad \theta = 45^\circ$ $\therefore 2\tan\theta + \cos^2\theta = 2 + \frac{1}{2} = \frac{5}{2}$	
4	$a_1 = 3, a_3 = 7$ $S_3 = \frac{3}{2}(3 + 7) = 15$	$1/_2$ $1/_2$
5	$\frac{AD}{DB} = \frac{AE}{EC} \qquad DE \parallel BC$ $\Longrightarrow \angle ADE = \angle ABC = 48^0$	$1/_2$ $1/_2$
6	4 places	1
	SECTION B	
7	HCF × LCM = Product of two numbers 9 × 360 = 45 × 2nd number 2nd number = 72	1 1
	OR	

	Let us assume, to the contrary that $7-\sqrt{5}$ is irrational $7-\sqrt{5} = \frac{p}{q}$, Where p & q are co-prime and q ≠ 0 $=\sqrt{5} = \frac{7q-p}{q}$ $\frac{7q-p}{q}$ is rational = $\sqrt{5}$ is rational which is a contradiction Hence $7-\sqrt{5}$ is irrational	1 1
8	20^{th} term from the end = $l-(n-1)d$ = 253-19 ×5 = 158	$^1/_2$ 1 $^1/_2$
	OR	
	$7a_7 = 11a_{11} \Rightarrow 7(a+6d) = 11(a+10d)$ $\Rightarrow a+17d = 0 \therefore a_{18} = 0$	1 1
9	$X = \frac{6-6}{5} = 0$ $Y = \frac{-10+15}{5} = 1$	1 1
10	Probability of either a red card or a queen $= \frac{26+2}{52} = \frac{28}{52}$ P(neither red car nor a queen) = $1-\frac{28}{52}$ $= \frac{24}{52}$ or $\frac{7}{13}$	1 1
11	Total number of outcomes = 36 Favourable outcomes are (1,2), (2,1), (1,3), (3,1), (1,5), (5,1) i.e. 6 Required probability = $\frac{6}{36}$ or $\frac{1}{6}$	1 1
12	For infinitely many solutions $\frac{p-3}{p} = \frac{3}{p} = \frac{-p}{-12}$ $\Rightarrow p^2-3p = 3p$ or $12\times3 = p^2$ $\Rightarrow p^2-6p = 0$ or $p = \pm 6$ p = 0,6 $\Rightarrow p = 6$	$^1/_2$ 1
	SECTION: C	
13	By Euclid's Division lemma 726 = 275 ×2+176 275 = 176 ×1 + 99 176 = 99 ×1+77 99 = 77 × 1+22 77=22 ×3 + 11 22 = 11 × 2+0 HCF = 11	6 × $^1/_2$ = 3

14	$5\sqrt{5}x^2+30x+8\sqrt{5}$ $= 5\sqrt{5}x^2+20x+10x+8\sqrt{5}$ $= 5x(\sqrt{5}x + 4)+2\sqrt{5}(\sqrt{5}x + 4)$ $= (\sqrt{5}x + 4)\ (5x+2\sqrt{5})$ Zeroes are $\frac{-4}{\sqrt{5}} = \frac{-4\sqrt{5}}{5}$ and $\frac{-2\sqrt{5}}{5}$	**1** **1** **1**
15	Let the speed of car at A be x km/h And the speed of car at B be y km/h Case 1 $\quad 8x-8y = 80$ $x-y = 10$ Case 2 $\quad \frac{4}{3}x + \frac{4}{3}y = 80$ $x+y = 60$ on solving x= 35 and y = 25 Hence, speed of cars at A and B are 35 km/h and 25 km/h respectively.	**1** **1** **1**
16	 Diagonals of parallelogram bisect each other $\Rightarrow$ midpoint of AC = midpoint of BD $\Rightarrow \left(\frac{1+k}{2}, \frac{-2+2}{2}\right) = \left(\frac{-4+2}{2}, \frac{-3+3}{2}\right)$ $\Rightarrow \frac{1+k}{2} = \frac{-2}{2}$ $\Rightarrow k = -3$	**$1\frac{1}{2}$** **$\frac{1}{2}$** 1
	OR	
	For collinearity of the points, area of the triangle formed by given Points is zero. $\Rightarrow \frac{1}{2}\{(3k-1)(k-7+k+2)+k(-k-2-k+2)+(k-1)(k-2-k+7)\} = 0$ $\Rightarrow \{(3k-1)(2k-5)-2k^2+5k-5\} = 0$ $\Rightarrow 4k^2-12k = 0$ $\Rightarrow k = 0\ ,\ 3$	**1** **1** **1**
17	LHS $= \cot\theta - \tan\theta$ $= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$ $= \frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}$ $= \frac{\cos^2\theta-1+\cos^2\theta}{\sin\theta\cos\theta}$ $= \frac{2\cos^2\theta-1}{\sin\theta\cos\theta}$ = RHS	**1** **$\frac{1}{2}$** **1** **$\frac{1}{2}$**
	OR	

$$\text{LHS} = \sin\theta(1+\tan\theta) + \cos\theta(1+\cot\theta)$$ **1**

$$= \sin\theta\left(1+\frac{\sin\theta}{\cos\theta}\right) + \cos\theta\left(1+\frac{\cos\theta}{\sin\theta}\right)$$

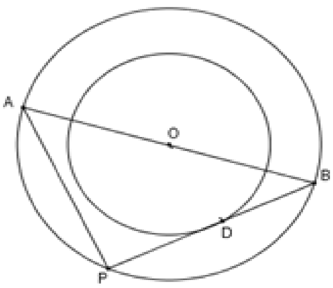
$$= \sin\theta\left(\frac{\cos\theta+\sin\theta}{\cos\theta}\right) + \cos\theta\left(\frac{\sin\theta+\cos\theta}{\sin\theta}\right)$$ **1**

$$= (\cos\theta + \sin\theta)\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$= \frac{\cos\theta+\sin\theta}{\cos\theta\sin\theta} = cosec\theta + sec\theta = \text{RHS}$$ **1**

SECTION: E

18



1

$\angle APB = 90^0$ (angle in semi-circle)

$\angle ODB = 90^0$ (radius is perpendicular to tangent) **$^1/_2$**

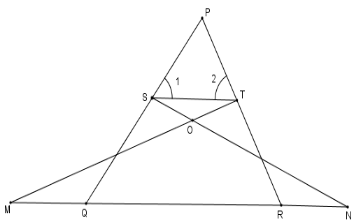
$\Delta ABP \sim \Delta OBD$

$\Rightarrow \frac{AB}{OB} = \frac{AP}{OD}$ **1**

$\Rightarrow \frac{26}{13} = \frac{AP}{8}$

$\Rightarrow$ AP = 16cm

19



$\angle 1 = \angle 2$ **1**

$\Rightarrow$ PT=PS …………………………………..………..(i)

$\Delta NSQ \cong \Delta MTR$

$\Rightarrow \angle NQS = \angle MRT$

$\Rightarrow \angle PQR = \angle PRQ$

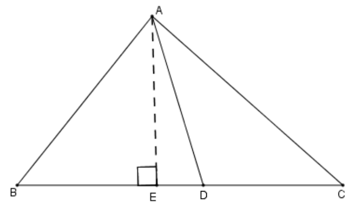
$\Rightarrow$ PR=PQ ……………………………………….(ii)

From (i) and (ii) $\frac{PT}{PR} = \frac{PS}{PQ}$ **1**

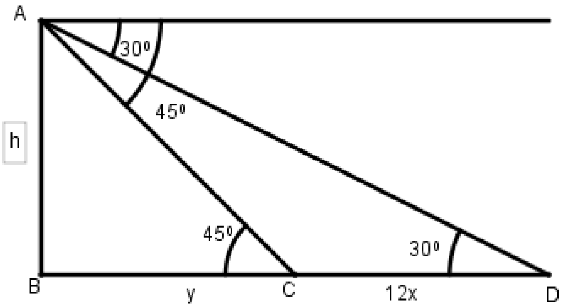
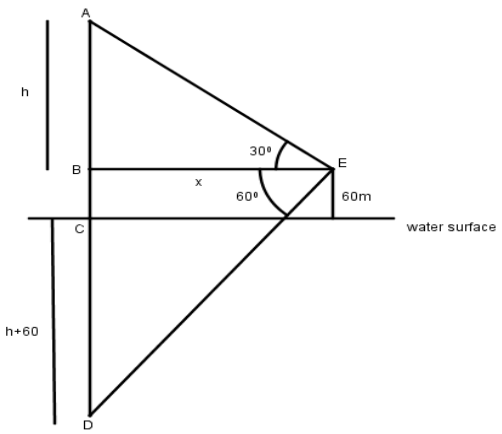
Also, $\angle TPS = \angle RPQ$ (common) **1**

$\Rightarrow \Delta PTS \sim \Delta PRQ$

OR

	 AD is median, So BD=DC. $\left.\begin{aligned} AB^2 &= AE^2+BE^2 \\ AC^2 &= AE^2+EC^2 \end{aligned}\right\}$ Adding both, $AB^2+AC^2 = 2AE^2+BE^2+CE^2$ $= 2(AD^2-ED^2)+(BD-ED)^2+(DC+ED)^2$ $= 2AD^2-2ED^2+BD^2+ED^2-2BD.ED+DC^2+ED^2+2CD.ED$ $= 2AD^2+BD^2+CD^2$ $= 2(AD^2+ BD^2)$	**1** **1** **1**
20	r = 42cm $\frac{2\pi r\theta}{360°} = 44$ $\theta = \frac{44\times360\times7}{2\times22\times42} = 60^0$ Area of minor segment = area of sector – area of corresponding triangle $= \frac{\pi r^2\theta}{360°} - \frac{\sqrt{3}}{4}r^2$ $= r^2[\frac{22}{7}\times\frac{60}{360}-\frac{\sqrt{3}}{4}]$ $= 42\times 42\,[\frac{11}{21}-\frac{\sqrt{3}}{4}]$ $= 42\times 42\ \times[\frac{44-21\sqrt{3}}{84}]$ $= 21\,(44-21\sqrt{3})\ cm^2$	**1** $^1/_2$ $^1/_2$ **1**
21	Volume of water flowing through pipe in 1 hour $= \frac{22}{7}\times 15\times 1000\times\frac{7}{100}\times\frac{7}{100}$ $= 231\ m^3$ Volume of rectangular tank $= 50\times 44\times\frac{21}{100}$ $= 22\times 21\ m^3$ Time taken to flow $231\ m^3$ of water = 1 hours ∴ Time taken to flow $22\times 21\ m^3$ of water $= \frac{1}{231}\times 22\times 21$ = 2 hours	**1** **1** **1**
	OR	
	Number of balls $= \frac{\text{Volume of solid sphere}}{\text{Volume of 1 spherical ball}}$ $= \frac{\frac{4}{3}\times\pi\times3\times3\times3}{\frac{4}{3}\times\pi\times0.3\times0.3\times0.3}$ = 1000	**1** **1** **1**

22	200-250 is the modal class Mode = $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$ $= 200 + \frac{12-5}{24-5-2} \times 50$ = 200+20.59 = Rs. 220.59	**1** **1** **$^1/_2$** **$^1/_2$**
	Section D	
23	Let the usual speed of the train be x km/h $\frac{300}{x} - \frac{300}{x+5} = 2$ $\Rightarrow$ $x^2+5x-750 = 0$ $\Rightarrow$ $(x+30)(x-25) = 0$ $\Rightarrow$ $x = -30, 25$ ∴ Usual Speed of the train = 25 km/h	**2** **1** **1**
	OR	
	$\frac{1}{(a+b+x)} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$ $\Rightarrow \frac{x-a-b-x}{x(a+b+x)} = \frac{b+a}{ab}$ $\Rightarrow -ab = x^2+(a+b)x$ $\Rightarrow x^2+ax+bx+ab = 0$ $\Rightarrow (x+a)(x+b) = 0$ $\Rightarrow x= -a,-b$	**1** **1** **1** **1**
24	n=50, a_3 =12 and a_{50} = 106 $\left.\begin{aligned} a+2d &= 12 \\ a+49d &= 106 \end{aligned}\right\}$ on solving, d=2 and a= 8 a_{29}= a+28d = 8+28×2 = 64	**$^1/_2$** **1** **1** **$^1/_2$** 1
25	Correct given, To prove, figure and construction Correct proof	**$^1/_2$ × 4 = 2** **2**
26	Correct construction of ΔABC Correct construction of similar triangle	**1** **3**

27	 Correct figure Let the speed of car be x m/ minutes In ΔABC, $\frac{h}{y} = \tan 45^0$ $\Rightarrow$ h = y In ΔABD, $\frac{h}{y+12x} = \tan 30^0$ $\Rightarrow h\sqrt{3} = y+12x$ $y\sqrt{3} - y = 12x$ $y = \frac{12x}{\sqrt{3}-1} = \frac{12x(\sqrt{3}+1)}{2}$ $\Rightarrow y = 6x(\sqrt{3}+1)$ Time taken from C to B = $6(\sqrt{3}+1)$ minutes	1 1 $^1/_2$ 1 $^1/_2$
	OR	
	 Correct figure In ΔABE, $\frac{h}{x} = \tan 30^0$ $\Rightarrow x = h\sqrt{3}$	1 1 $^1/_2$ 1 $^1/_2$

	In ΔBDE, $\frac{h+60+60}{x} = \tan 60^0$ $h+120 = x\sqrt{3}$ $h+120 = h\sqrt{3} \times \sqrt{3}$ $2h = 120$ $h = 60$ $\therefore height\ of\ cloud\ from\ surface\ of\ water = (60+60)m = 120m$	
28	(table below)	**1**

Class Interval	Frequency	cf
0-100	2	2
100-200	5	7
200-300	x	7+x
300-400	12	19+x
400-500	17	36+x
500-600	20	56+x
600-700	y	56+x+y
700-800	9	65+x+y
800-900	7	72+x+y
900-1000	4	76+x+y

N=100 $\Rightarrow$ 76+x+y =100 **$^1/_2$**

$\Rightarrow$ x+y = 24..(i)

Median = 525 $\Rightarrow$ 500 – 600 is median class **$^1/_2$**

60-80 is the median class 1

Median = $l + \frac{\frac{n}{2}-cf}{f} \times h$

$\Rightarrow 500 + \left(\frac{50-36-x)}{20}\right) \times 100 = 525$ 1

$\Rightarrow (14-x) \times 5 = 25$

$\Rightarrow$ x = 9

$\Rightarrow$ from (1), y = 5.96

OR

Marks	Number of students	cf
0-10	5	5
10-20	3	8
20-30	4	12
30-40	3	15
40-50	3	18
50-60	4	22
60-70	7	29
70-80	9	38
80-90	7	45
90-100	8	53

Correct table **1**
Drawing correct Ogive **2**
Median=64 **1**

29

$r_1 = 15\text{cm}$, $r_2 = 5\text{cm}$

$h = 24\text{cm}$

$l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{24^2 + 10^2} = 26\text{cm}$ **1**

Curved surface area of bucket $= \pi(r_1 + r_2)l$ **1**

$= \frac{22}{7} \times (15 + 5) \times 26$

$= \frac{22 \times 20 \times 26}{7}$ **1**

$= \frac{11440}{7} \text{cm}^2$ or 1634.3cm^2 **1**

30

1. $Sec\theta + tan\theta = p$

$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = p$

$1+\sin\theta = p\cos\theta$

$= p\sqrt{1 - \sin^2\theta}$

$(1+\sin\theta)^2 = p^2(1 - \sin^2\theta)$

$1 + sin^2\theta + 2sin\theta = p^2 - p^2\sin^2\theta$

$(1 + p^2)\, sin^2\theta + 2sin\theta + (1 - p^2) = 0$ **1**

$D = 4 - 4(1+p^2)(1-p^2)$

$= 4 - 4(1-p^4) = 4p^4$ **1**

$\sin\theta = \frac{-2 \pm \sqrt{4p^4}}{2(1+p^2)} = \frac{-1 \pm p^2}{(1+p^2)}$ **1/2**

$= \frac{p^2-1}{p^2+1}, -1$

$\therefore Cosec\,\theta = \frac{p^2+1}{p^2-1}, -1$ **1**